

AWM 11 – UNIT 4 – TRIGONOMETRY OF RIGHT TRIANGLES

Assignment	Title	Work to complete	Complete
1	<i>Triangles</i>	Labelling Triangles	
2	<i>Pythagorean Theorem</i>	Exploring Pythagorean Theorem	
3	<i>More Pythagorean Theorem</i>	Using Pythagorean Theorem in Problem Solving	
4	<i>Pythagorean Triples</i>	Pythagorean Triples	
5	<i>Trigonometry Review</i>	Trigonometry	
	Quiz 1		
6	<i>The Trigonometric Ratios: The Sine Ratio The Cosine Ratio The Tangent Ratio</i>	The Trigonometric Ratios	
7	<i>Using The Sine Ratio Using The Cosine Ratio Using The Tangent Ratio</i>	Finding Sides In Right Triangles	
	Quiz 2		
8	<i>Angle of Elevation and Depression</i>	Angle of Elevation and Depression	
9	<i>The Trigonometric Ratios</i>	Word Problems	
10	<i>Finding Angles in Right Triangles</i>	Finding Angles in Right Triangles	
	Quiz 3		
11	<i>Solving Complex Problems</i>	Working with Two or More Triangles	
12	<i>Solving Complex 3D Problems in the Real World</i>	Working with Triangles in 3-D	
Practice Test	Practice Test How are you doing?	Get this page from your teacher	
Math Journal	Math Journal	Journal entry based on criteria on handout and question jointly chosen.	
Self-Assessment	Self-Assessment	On the next page, complete the self-assessment assignment.	
Unit Test	Unit Test Show me your stuff!		

Self-Assessment

In the following chart, show how confident you feel about each statement by drawing one of the following: 😊, 😐, or ☹️. Then discuss this with your teacher **BEFORE** you write the test!

Statement	😊 😐 ☹️
After completing this chapter;	
<ul style="list-style-type: none"> I can use the Pythagorean theorem to calculate the missing side of a right triangle 	
<ul style="list-style-type: none"> I know when to choose sine (sin), cosine (cos) or tangent(tan) based on the information given 	
<ul style="list-style-type: none"> I can use the three basic trigonometric functions (sin, cos, tan) to find a missing side or angle of a right triangle 	
<ul style="list-style-type: none"> I can use the three basic trigonometric ratios to solve problems involving two or three triangles 	
<ul style="list-style-type: none"> I can determine the angle of elevation and the angle of depression from words or a diagram, and use them with the trigonometric ratios 	
<ul style="list-style-type: none"> I can use the three basic trigonometric ratios to solve problems in both 2-D and 3-D contexts 	
<ul style="list-style-type: none"> I can determine if the solutions that I find are reasonable based on my knowledge of the lengths of the sides in a triangle 	

Vocabulary: Unit 4

angle of depression

angle of elevation

cosine

hypotenuse

leg

right triangle

sine

tangent

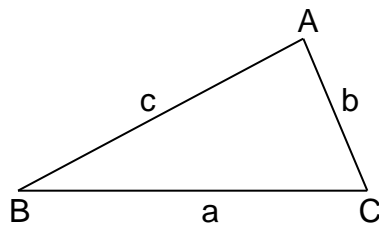
TRIANGLES

In this unit, you will be looking at triangles, specifically right angle triangles, also called right triangles. You will learn about Pythagorean Theorem and the basic trigonometric ratios. But first it is necessary to start with some facts about triangles.

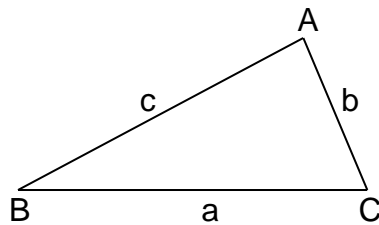
Fact 1: Every triangle contains 3 sides and 3 angles or vertices (plural of vertex).

Fact 2: The measurements of these angles always total 180° . Remember this from the last unit??

Fact 3: To identify the side or vertex in a triangle, it is important to label the triangle following a standard routine. Each vertex of a triangle is labeled with a capital case letter – like “A” - and each side is labeled with the lower case letter that matches the opposite vertex. An example is below.



Another way to label the sides is with the capital letters of the two vertices the side connects. An example is below.

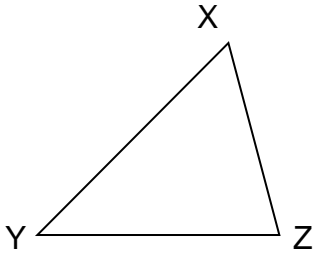


Side a can be called BC.
Side b can be called AC.
Side c can be called AB.

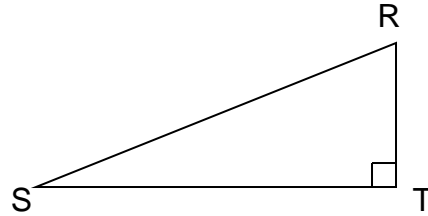
ASSIGNMENT 1 – LABELLING TRIANGLES

1) Label each side of the triangles below using a single lower case letter matching the opposite vertex.

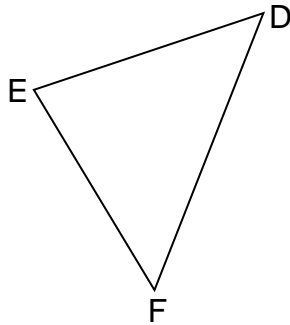
a)



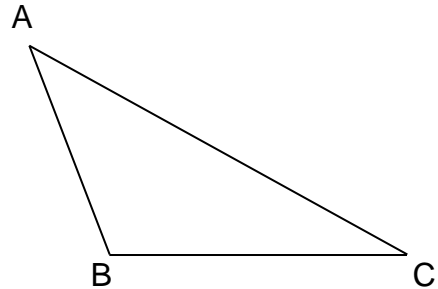
b)



c)

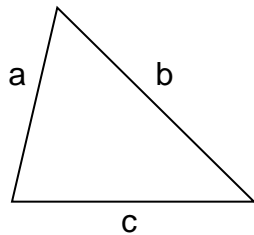


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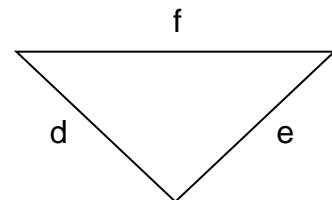


2) Label each vertex of the triangles below using a single capital letter matching the opposite side.

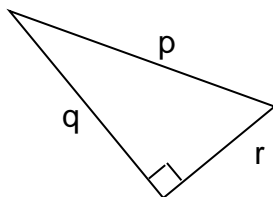
a)



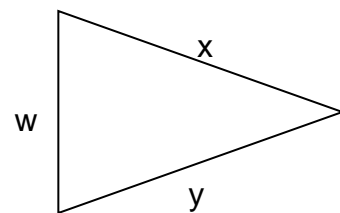
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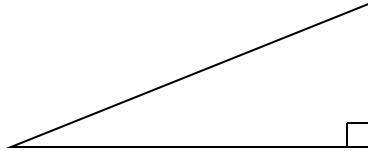
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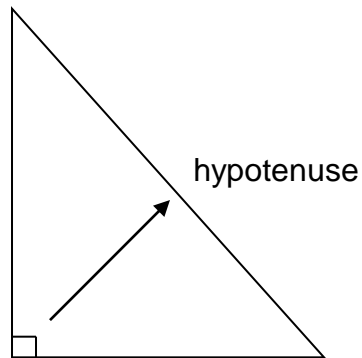
PYTHAGOREAN THEOREM

Pythagorean Theorem states the relationship between the sides of a right triangle. So, more facts about triangles are necessary.

Fact 4: A triangle that contains a 90° angle (a right angle) is called a right triangle (or right-angle triangle).



Fact 5: The side of the triangle that is opposite the 90° angle is always called the **hypotenuse**. It is labelled in the triangle below. The other two sides of the triangle are called legs.

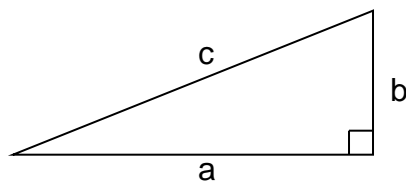


Fact 6: The hypotenuse is always the longest side in the triangle. It is always opposite the largest angle which is the 90° or right angle.

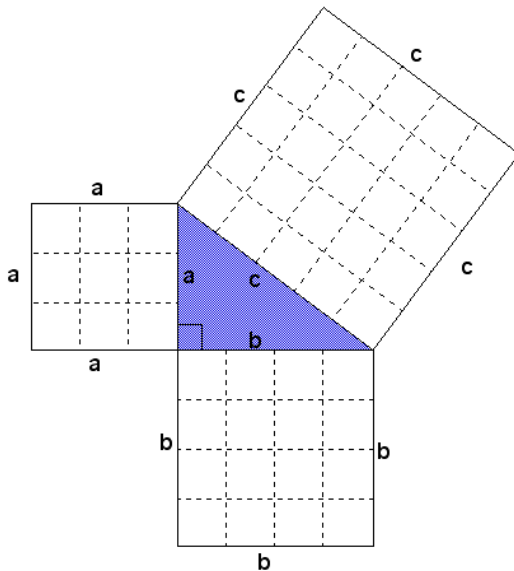
Fact 7: Pythagorean Theorem states that in any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. So in $\triangle ABC$ with the right angle at C, the following relationship is true:

$$c^2 = a^2 + b^2$$

where a and b are the other 2 legs of the triangle.



Often Pythagorean Theorem is illustrated as the square of the sides as follows:



Notice that the length of side a is 3 boxes, side b is 4 boxes, and side c is 5 boxes. So if we calculate the area of each square, the following is true:

$$\begin{aligned} c \times c &= c^2 = 5 \times 5 = 25 \\ b \times b &= b^2 = 4 \times 4 = 16 \\ a \times a &= a^2 = 3 \times 3 = 9 \end{aligned}$$

And we know that

$$c^2 = a^2 + b^2$$

So $25 = 9 + 16$ which is a true statement!

We can also rearrange the equation to find the length one of the legs;

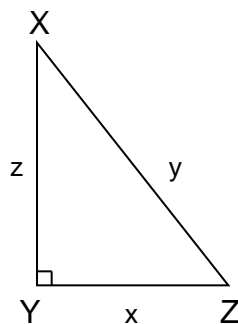
$$\begin{aligned} c^2 &= a^2 + b^2 \\ a^2 &= c^2 - b^2 \\ b^2 &= c^2 - a^2 \end{aligned}$$

When we use Pythagorean Theorem to find a length of the hypotenuse or a leg, you need to have a calculator that has the square root function $\sqrt{\quad}$ on it. The computer symbol looks like this: $\sqrt{\quad}$ or $\sqrt{\quad}$

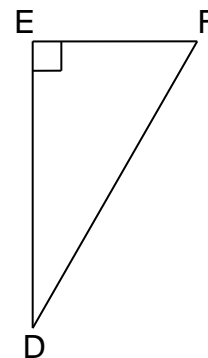
ASSIGNMENT 2 – EXPLORING PYTHAGOREAN THEOREM

1) Using the following triangles, use lettering provided to state the Pythagorean relations that apply.

a)



b)



2) Find the missing value in each of the following to 2 decimal places. Remember to use the square root to solve for each final side length. An example is done for you.

Example: $b^2 = 5^2 + 12^2$
 $b^2 = 25 + 144$
 $b^2 = 169$
 $b = \sqrt{169} = 13$

a) $p^2 = 6^2 + 9^2$

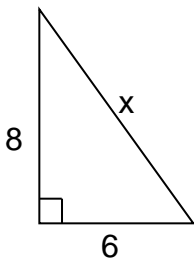
b) $m^2 = 4^2 + 7^2$

c) $y^2 = 8^2 - 5^2$

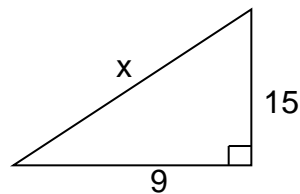
d) $z^2 = 10^2 - 5^2$

3) Calculate the missing side length (x) in each triangle to 1 decimal place (as needed).

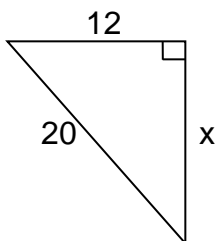
a)



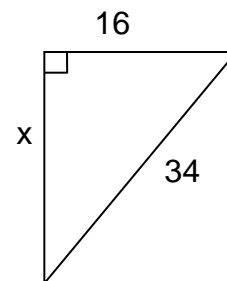
b)



c)



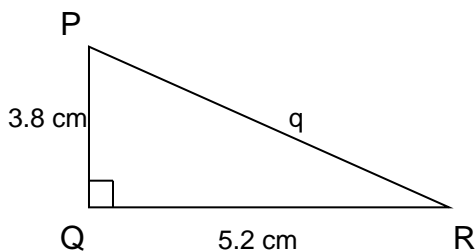
d)



MORE PYTHAGOREAN THEOREM

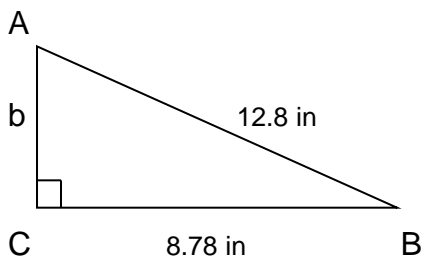
Pythagorean Theorem can be used to solve for an unknown side in a right triangle. It is important to recognize if you are trying to solve for the hypotenuse or one of the legs. If you are solving for the hypotenuse, you will add the squares of the other two sides, getting a larger side length (the hypotenuse is the longest side length). But if you are given the hypotenuse, subtract the square of the leg from the square of the hypotenuse in order to get a smaller side (than the hypotenuse).

Example 1: Use Pythagorean Theorem to find the length of the missing side to one decimal place.



Solution: $q^2 = p^2 + r^2$
 $q^2 = 5.2^2 + 3.8^2$
 $q^2 = 27.04 + 14.44$
 $q^2 = 41.48$
 $\sqrt{q^2} = \sqrt{41.48}$
 $q \approx 6.44$ cm
Side q is approximately 6.4 cm

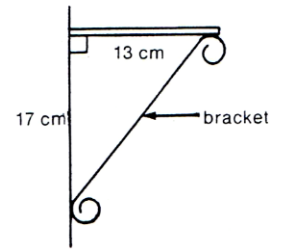
Example 2: Use Pythagorean Theorem to find the length of the missing side to one decimal place.



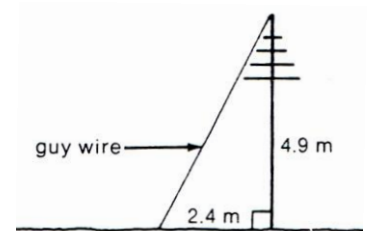
Solution: $c^2 = a^2 + b^2$
So, $b^2 = c^2 - a^2$
 $b^2 = 12.8^2 - 8.78^2$
 $b^2 = 163.84 - 77.0884$
 $b^2 = 86.7516$
 $b = \sqrt{86.7516}$
 $b \approx 9.31$ cm
Side b is approximately 9.31 cm

ASSIGNMENT 3 – USING PYTHAGOREAN THEOREM IN PROBLEM SOLVING

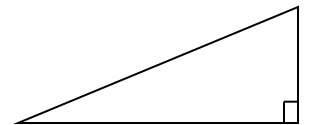
1) Solve for the length of the bracket in the picture, to one decimal place.



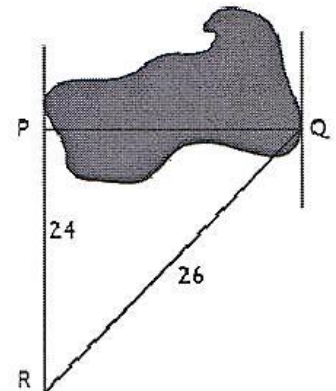
2) What is the length of the guy wire in the picture below, to one decimal place?



3) A ramp into a house rises up (\uparrow) 3.5 meters over a horizontal distance (\leftrightarrow) of 10.5 metres. How long is the ramp? Use the triangle below and show your work.



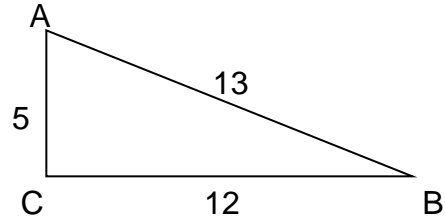
4) You need to find the width of a lake, **PQ**, as shown. The measurements of the other sides are given on the diagram. You are certain that $\angle P = 90^\circ$. What is the width of the lake?



- 5) A ladder is leaned against a house. The base of the ladder is d feet away from the house. Draw a diagram and then write the Pythagorean relationship that exists for these lengths. Use ℓ for the ladder, h for the house, and d for the distance the ladder is from the house. You are not required to solve this question.
- 6) A 40 foot ladder reaches 38 feet up the side of a house. How far from the side of the house is the base of the ladder? Draw a diagram and show your work.
- 7) A flagpole is 12 metres tall. It makes a shadow on the ground that is 15 metres long. How long is a line that joins the top of the flagpole with the end of the shadow? Draw a diagram and show your work.
- 8) The size of a flat screen TV is given by the length of the diagonal of the screen (joining opposite corners). What is the length of a diagonal if the sides are 40.5 inches by 22.625 inches? Draw a diagram and show your work.

PYTHAGOREAN TRIPLES

A **Pythagorean Triple** is a set of three numbers that satisfy the Pythagorean Theorem and are all whole numbers (no decimals). If a set of numbers satisfies the Pythagorean relationship, then the triangle must be a right triangle. An example is shown below.



$$\begin{aligned}\text{Left Side} &= c^2 \\ &= 13^2 \\ &= \mathbf{169}\end{aligned}$$

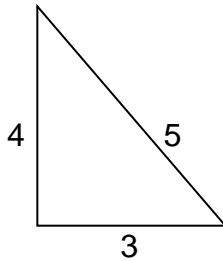
$$\begin{aligned}\text{Right Side} &= a^2 + b^2 \\ &= 12^2 + 5^2 \\ &= 144 + 25 \\ &= \mathbf{169}\end{aligned}$$

Therefore, Left Side = Right Side. This triangle is a right angle triangle, and the set of numbers, 5, 12, and 13 are a **Pythagorean Triple**.

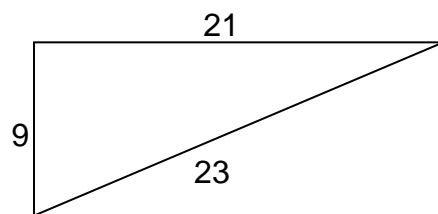
ASSIGNMENT 4 – PYTHAGOREAN TRIPLES

1) Which of the following triangles are right triangles? Show your work as proof.

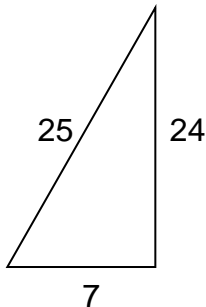
a)



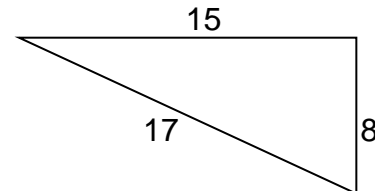
b)



c)



d)



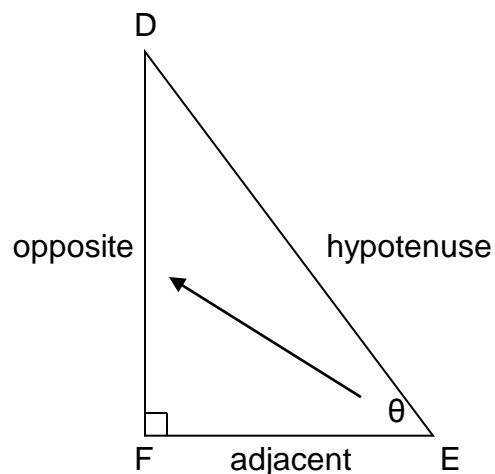
TRIGONOMETRY REVIEW

Trigonometry is one of the most important topics in mathematics. Trigonometry is used in many fields including engineering, architecture, surveying, aviation, navigation, carpentry, forestry, and computer graphics. Also, until satellites, the most accurate maps were constructed using trigonometry.

The word **trigonometry** means triangle measurements. It is necessary to finish our triangle facts here.

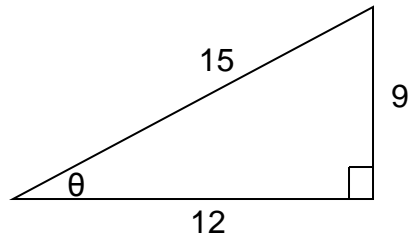
Fact 8: In trigonometry, the other two sides (or legs) of the triangle are referred to as the **opposite** and **adjacent** sides, depending on their relationship to the angle of interest in the triangle.

In this example, if we pick angle DEF – the angle labelled with the Greek letter θ called ***theta*** – then we are able to distinguish the sides as illustrated in the diagram below.



The side that is opposite the angle of interest, in this case θ , is called the **opposite** side. The side that is nearest to angle θ and makes up part of the angle is called the **adjacent** side. To help you, remember that adjacent means beside. Although the hypotenuse occupies one of the two adjacent positions, it is **never** called the adjacent side. It simply remains the hypotenuse. This is why it is identified first. It is recommended to label the side in the order hypotenuse, opposite, and finally adjacent. You may use initials for these side, h, o, and a, but always use lower case letters to avoid mixing up the labelling with a vertex.

Example 1: Using the triangle below, answer the questions.



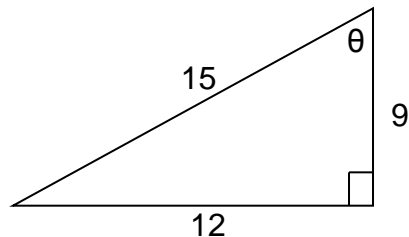
- 1) What is the hypotenuse? _____
- 2) What is the opposite side to θ ? _____
- 3) What is the adjacent side to θ ? _____

Solution:

- 1) What is the hypotenuse? 15
- 2) What is the opposite side to θ ? 9
- 3) What is the adjacent side to θ ? 12

This example uses the same triangle as in Example 1; however, this time, the other acute angle is labelled as θ . This is done to show that the opposite and adjacent sides switch when the other angle is the angle of interest. The hypotenuse **always** stays the same.

Example 2: Using the triangle below, answer the questions.



- 1) What is the hypotenuse? _____
- 2) What is the opposite side to θ ? _____
- 3) What is the adjacent side to θ ? _____

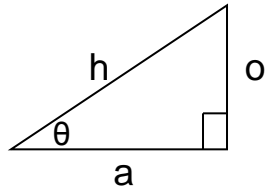
Solution:

- 1) What is the hypotenuse? 15
- 2) What is the opposite side to θ ? 12
- 3) What is the adjacent side to θ ? 9

ASSIGNMENT 5 – TRIGONOMETRY

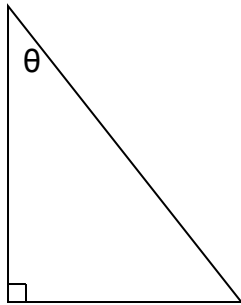
For each of the right triangles below, mark the hypotenuse, and the sides that are opposite and adjacent sides to θ as shown in the example.

Example:

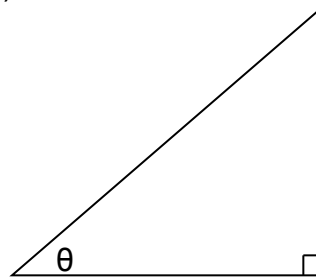


h = hypotenuse
o = opposite
a = adjacent

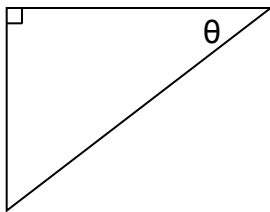
1)



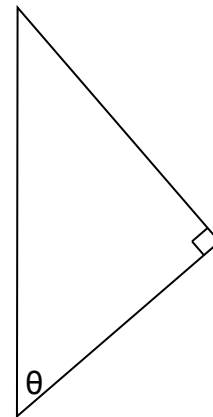
2)



3)



4)



ASK YOUR TEACHER FOR QUIZ 1

TRIGONOMETRIC RATIOS

In the previous unit about similar figures, you learned that the ratios of corresponding sides of similar triangles are equal. When the angles of different triangles are the same, the ratio of the sides within the triangle will always be the same. They depend only on the measure of the angle of interest, not the size of the triangle. These ratios are the trigonometric ratios.

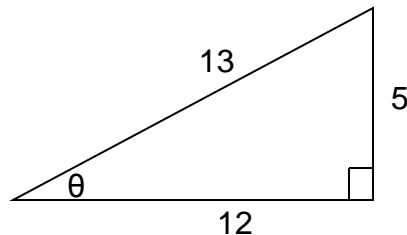
There are three trigonometric ratios we are concerned with: sine, cosine, and tangent.

THE SINE RATIO

The *sine of angle θ* means the ratio of the length of opposite side to the length of the hypotenuse. It is abbreviated as **sin θ** but read as sine θ . It is written like this:

$$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}} \quad \text{or} \quad \sin \theta = \frac{o}{h}$$

Example 1: Find the sine of θ in this triangle. Round to 4 decimal places.



Solution:

The opposite side is 5 and the hypotenuse is 13. So

$$\sin \theta = \frac{o}{h} = \frac{5}{13} = 0.3846 \quad \text{So} \quad \sin \theta = 0.3846$$

Note: Rounding to 4 decimal places is standard when calculating trigonometric ratios.

Example 2: Use your calculator to determine the following sine ratios. Round to 4 decimal places.

- a) $\sin 15^\circ$ b) $\sin 67^\circ$ c) $\sin 42^\circ$

******* REMEMBER TO SET YOUR CALCULATOR ON DEGREES (DEG) *******

Solution: Type “sin” followed by the angle, and then “=” to solve

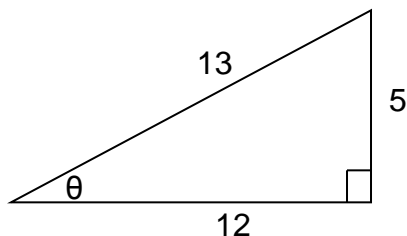
- a) $\sin 15^\circ = 0.2588$ b) $\sin 67^\circ = 0.9205$ c) $\sin 42^\circ = 0.6691$

THE COSINE RATIO

The *cosine of angle θ* means the ratio of the adjacent side to the hypotenuse. It is abbreviated as **cos θ** but read as cosine θ . It is written like this:

$$\cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}} \quad \text{or} \quad \cos \theta = \frac{a}{h}$$

Example 1: Find the cosine of θ in this triangle.



Solution:

The adjacent side is 12 and the hypotenuse is 13. So

$$\cos \theta = \frac{a}{h} = \frac{12}{13} = 0.9231$$

Note: Rounding to 4 decimal places is standard when calculating trigonometric ratios.

Example 2: Use your calculator to determine the following cosine ratios. Round to 4 decimal places.

- a) $\cos 15^\circ$ b) $\cos 67^\circ$ c) $\cos 42^\circ$

******* REMEMBER TO SET YOUR CALCULATOR ON DEGREES (DEG) *******

Solution: Type “cos” followed by the angle, and then “=” to solve

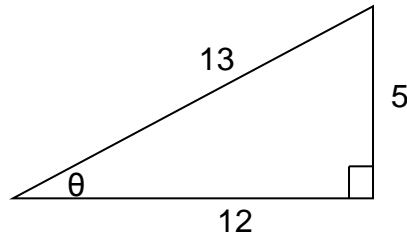
- a) $\cos 15^\circ = 0.9659$ b) $\cos 67^\circ = 0.3907$ c) $\cos 42^\circ = 0.7431$

THE TANGENT RATIO

The *tangent of angle θ* means the ratio of the opposite side to the adjacent side. It is abbreviated as **tan θ** but read as tangent θ . It is written like this:

$$\tan \theta = \frac{\textit{opposite}}{\textit{adjacent}} \quad \text{or} \quad \tan \theta = \frac{o}{a}$$

Example 1: Find the tangent of θ in this triangle.



Solution:

The opposite side is 5 and the adjacent side is 12. So

$$\tan \theta = \frac{o}{a} = \frac{5}{12} = 0.4167$$

Note: Rounding to 4 decimal places is standard when calculating trigonometric ratios.

Example 2: Use your calculator to determine the following tangent ratios. Round to 4 decimal places.

- a) $\tan 15^\circ$ b) $\tan 67^\circ$ c) $\tan 42^\circ$

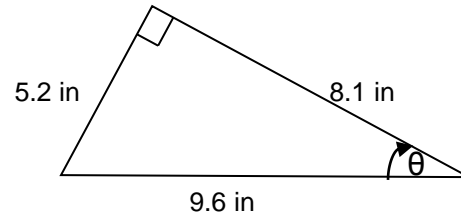
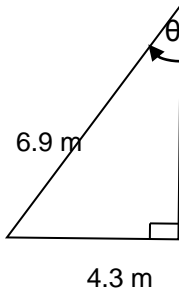
******* REMEMBER TO SET YOUR CALCULATOR ON DEGREES (DEG) *******

Solution: Type “tan” followed by the angle, and then “=” to solve

- a) $\tan 15^\circ = 0.2679$ b) $\tan 67^\circ = 2.3559$ c) $\tan 42^\circ = 0.9004$

ASSIGNMENT 6 – THE TRIGONOMETRIC RATIOS

1) Calculate the value of **sin θ** to four decimal places.



2) Use your calculator to determine the value of each of the following sine ratios to four decimal places.

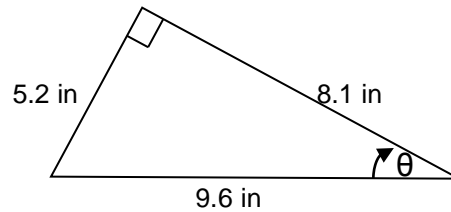
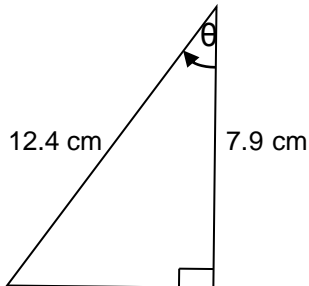
a) $\sin 10^\circ =$ _____

b) $\sin 48^\circ =$ _____

c) $\sin 77^\circ =$ _____

d) $\sin 85^\circ =$ _____

3) Calculate the value of **cos θ** to four decimal places.



4) Use your calculator to determine the value of each of the following cosine ratios to four decimal places.

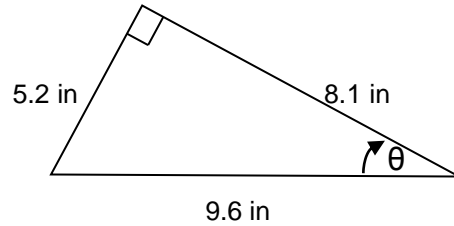
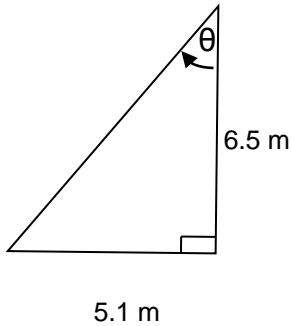
e) $\cos 10^\circ =$ _____

f) $\cos 48^\circ =$ _____

g) $\cos 77^\circ =$ _____

h) $\cos 85^\circ =$ _____

5) Calculate the value of **tan θ** to four decimal places.



6) Use your calculator to determine the value of each of the following tangent ratios to four decimal places.

i) $\tan 10^\circ =$ _____

j) $\tan 48^\circ =$ _____

k) $\tan 77^\circ =$ _____

l) $\tan 85^\circ =$ _____

7) There are two special sine ratios. Calculate the following.

a) $\sin 0^\circ =$ _____

b) $\sin 90^\circ =$ _____

8) There are two special cosine ratios. Calculate the following.

a) $\cos 0^\circ =$ _____

b) $\cos 90^\circ =$ _____

9) There are some special tangent ratios. Calculate the following.

a) $\tan 0^\circ =$ _____

b) $\tan 45^\circ =$ _____

c) $\tan 89^\circ =$ _____

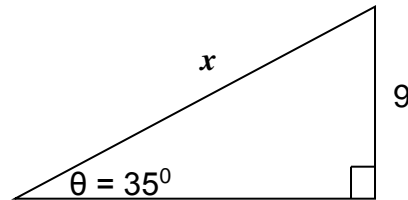
d) $\tan 90^\circ =$ _____

Note what the answer key says that the tan 90° equals. Just because your calculator says one thing, doesn't mean the calculator knows what is going on!

USING THE SINE RATIO

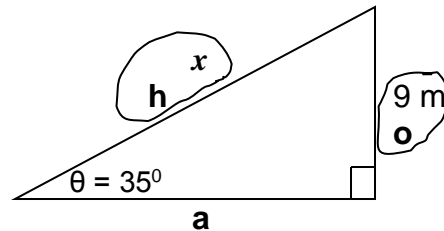
Whenever one side and one angle of a right triangle are already known, the remaining sides can be found using the trigonometric ratios. The sine ratio can be used to find missing parts of a right triangle.

Example 1: Use the sine ratio to find the x in the triangle below.



Solution:

Step 1: Label the sides of the triangle with **h**, **o** and **a**



Step 2: Circle the number with the side it represents and the unknown (x) with the side it represents.

Step 3: Identify the ratio required to solve for x

Since **o** and **h** are being used, the correct ratio is **sin θ**

Step 4: Substitute the correct values into the correct ratio.

$$\sin \theta = \frac{o}{h}$$

$$\sin 35^\circ = \frac{9}{x}$$

Step 5: Solve using the process Cross Multiply and Divide.

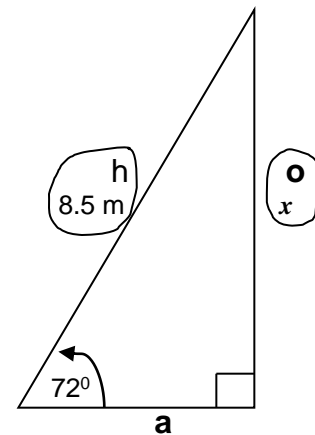
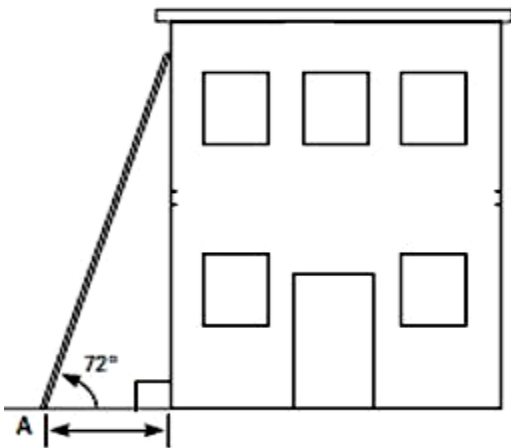
$$\text{Since } \sin 35^\circ = \frac{\sin 35}{1}, \text{ then } \sin 35^\circ = \frac{9}{x} \text{ becomes } \frac{\sin 35}{1} = \frac{9}{x}$$

$$\begin{aligned} x &= 9 \times 1 \div \sin 35^\circ \\ &= 15.7 \text{ m} \end{aligned}$$

Example 2: A ladder 8.5 m long makes an angle of 72° with the ground. How far up the side of a building will the ladder reach?

Solution:

Sketch a diagram and place the information from the question on this diagram. Remember that there will always be a right triangle in your diagram. It is often helpful to draw that triangle and copy the key information from the sketch.



Step 1: Label the sides of the triangle with **h**, **o** and **a**
See above right.

Step 2: Circle the number with the side it represents and the unknown (x) with the side it represents.

Step 3: Identify the ratio required to solve for x

Since **o** and **h** are being used, the correct ratio is **sin θ**

Step 4: Substitute the correct values into the correct ratio.

$$\sin \theta = \frac{o}{h}$$

$$\sin 72^\circ = \frac{x}{8.5}$$

Step 5: Solve using the process Cross Multiply and Divide.

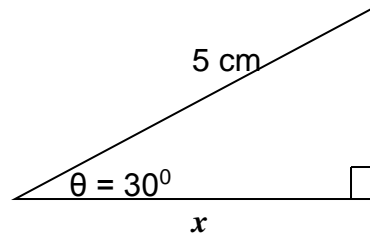
$$\text{Since } \sin 72^\circ = \frac{\sin 72}{1}, \text{ then } \sin 72^\circ = \frac{x}{8.5} \text{ becomes } \frac{\sin 72}{1} = \frac{x}{8.5}$$

$$\begin{aligned} x &= \sin 72^\circ \times 8.5 \div 1 \\ &= 8.1 \text{ m} \end{aligned}$$

USING THE COSINE RATIO

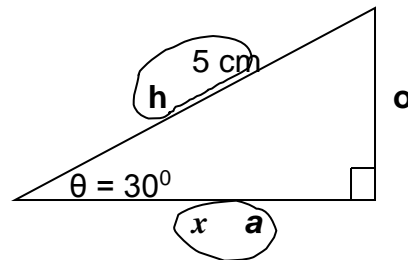
Whenever one side and one angle of a right triangle are already known, the remaining sides can be found using the trigonometric ratios. The cosine ratio can be used to find missing parts of a right triangle.

Example 1: Use the correct trig ratio to find the x in the triangle below.



Solution:

Step 1: Label the sides of the triangle with **h**, **o** and **a**



Step 2: Circle the number with the side it represents and the unknown (x) with the side it represents.

Step 3: Identify the ratio required to solve for x

Since **a** and **h** are being used, the correct ratio is **cos θ**

Step 4: Write down the chosen ratio and substitute the correct values into the correct ratio.

$$\cos \theta = \frac{a}{h}$$

$$\cos 30^\circ = \frac{x}{5}$$

Step 5: Solve using the process Cross Multiply and Divide.

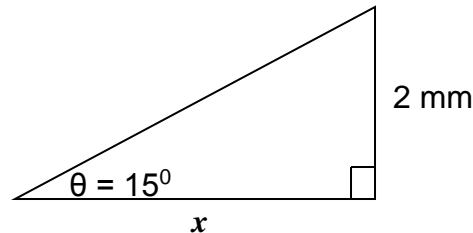
$$\text{Since } \cos 30^\circ = \frac{\cos 30}{1}, \text{ then } \cos 30^\circ = \frac{x}{5} \text{ becomes } \frac{\cos 30}{1} = \frac{x}{5}$$

$$\begin{aligned} x &= \cos 30^\circ \times 5 \div 1 \\ &= 4.3 \text{ cm} \end{aligned}$$

USING THE TANGENT RATIO

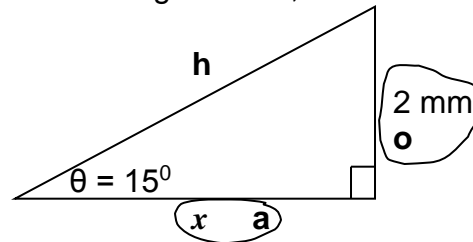
Whenever one side and one angle of a right triangle are already known, the remaining sides can be found using the trigonometric ratios. The tangent ratio can be used to find missing parts of a right triangle.

Example 1: Use the correct trig ratio to find the x in the triangle below.



Solution:

Step 1: Label the sides of the triangle with **h**, **o** and **a**



Step 2: Circle the number with the side it represents and the unknown (x) with the side it represents.

Step 3: Identify the ratio required to solve for x

Since **o** and **a** are being used, the correct ratio is **tan θ**

Step 4: Substitute the correct values into the correct ratio.

$$\tan \theta = \frac{o}{a}$$

$$\tan 15 = \frac{2}{x}$$

Step 5: Solve using the process Cross Multiply and Divide.

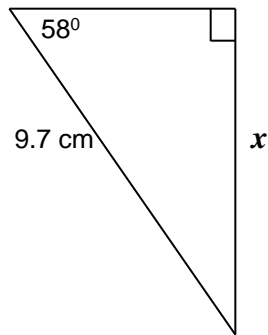
$$\text{Since } \tan 15^\circ = \frac{\tan 15}{1}, \text{ then } \tan 15^\circ = \frac{2}{x} \text{ becomes } \frac{\tan 15}{1} = \frac{2}{x}$$

$$\begin{aligned} x &= 2 \times 1 \div \tan 15^\circ \\ &= 7.5 \text{ mm} \end{aligned}$$

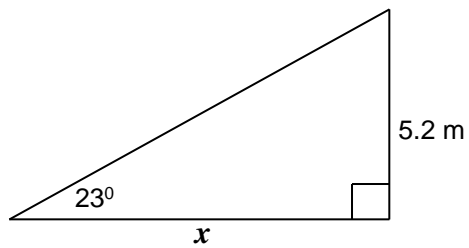
ASSIGNMENT 7 – FINDING SIDES IN RIGHT TRIANGLES

1) Calculate the length of the side indicated in the following diagrams. Round to **one** decimal place. **SHOW ALL STEPS AND WORK!!!** Check that your calculator is on degrees “DEG”.

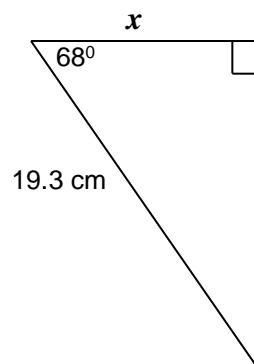
a)



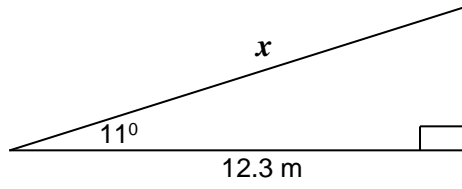
b)



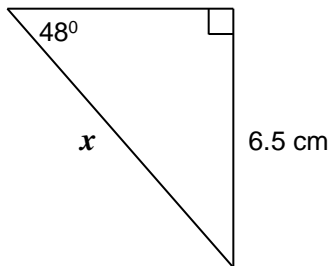
c)



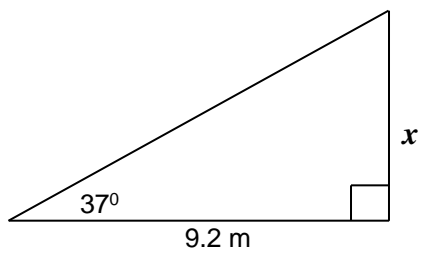
d)



e)



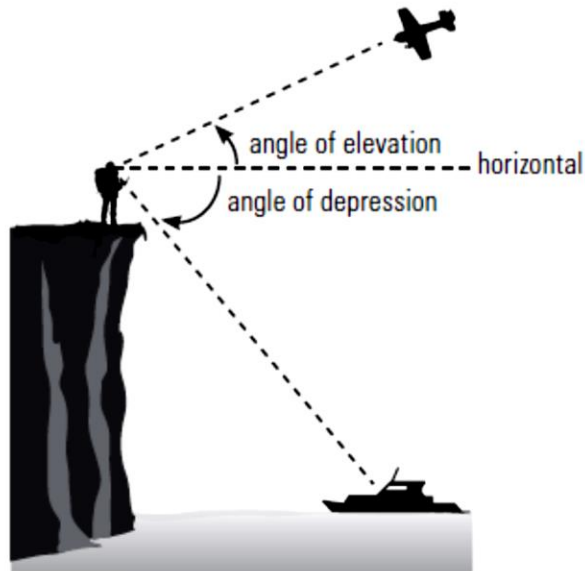
f)



ASK YOUR TEACHER FOR QUIZ 2

ANGLE OF ELEVATION AND DEPRESSION

When you look up at an airplane flying overhead for example, the angle between the horizontal and your line of sight is called the angle of elevation.

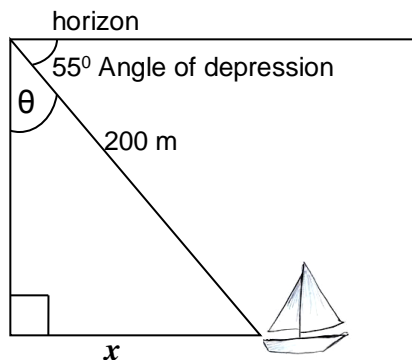


When you look down from a cliff to a boat passing by, the angle between the horizontal and your line of sight is called the angle of depression.

When you are given the angle of depression, it is important to carefully use this angle in your triangle.

Example 1: You are standing at the top of a cliff. You spot a boat 200 m away at an angle of depression of 55° to the horizon. How far is the boat from the coast? Draw a diagram to illustrate this situation.

Solution: Draw a diagram, label it with the information, and then solve the triangle.



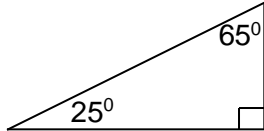
The angle inside the triangle is the complement to the angle of depression. To find that angle, do the following:
 $\theta = 90^\circ - 55^\circ$
 $\theta = 35^\circ$

ASSIGNMENT 8 – ANGLE OF ELEVATION AND DEPRESSION

Check that your calculator is on degrees “DEG”.

Using the angle of elevation and the angle of depression correctly in triangles is a challenging concept. If you are having difficulties with these next 2 pages, make sure you discuss it with your teacher.

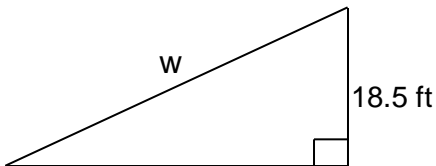
1) In the triangle below, what is the measure of the angle of elevation?



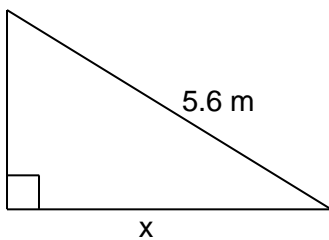
Measure _____

2) Write the degree measurement of the angle of elevation in each diagram. Then find the length of the unknown side, to one decimal place. Show all work.

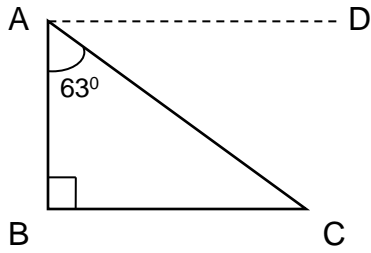
a) Angle of elevation = 43°



b) Angle of elevation = 21°



3) In the diagram below, what is the measure of the angle of depression? How do we name the angle of depression in this diagram? (See page 3 if you need a review.)

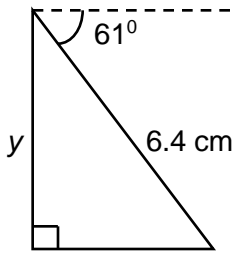


Measure _____

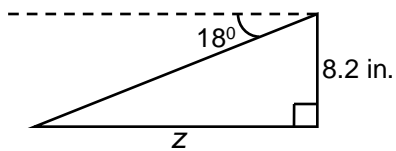
Name _____

4) Write a degree measurement in the triangle for each diagram. Then find the length of the unknown side, to one decimal place. Show all work.

a)

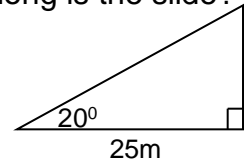


b)

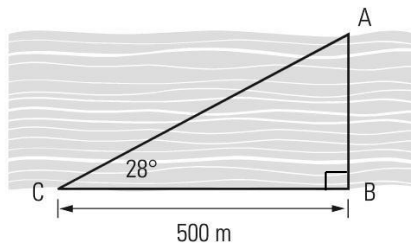


ASSIGNMENT 9 – WORD PROBLEMS

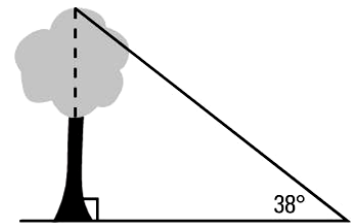
- 1) A child's slide rises to a platform at the top. If the angle of elevation of the slide is 20° , and the horizontal distance that the slide covers is 25 m long, how long is the slide?



- 2) A surveyor must determine the distance AB across a river. If he knows the information in the diagram below, how wide is the river?

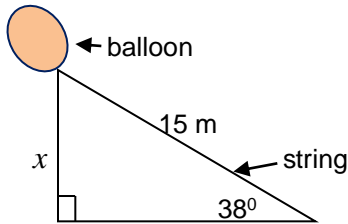


- 3) A tree is measured to be 12.4 m tall. If a man views the top of the tree at an angle of elevation of 38° , how far away from the tree is he standing?



- 4) A ladder is placed against the side of the house. If the base is 41 feet away from the house, and the angle of elevation between the ladder and the ground is 70° , how long is the ladder?

- 5) A weather balloon, which is blowing in the wind, is tied to the ground with a 15 m string. How high is the balloon (x) if the angle of elevation is 38° ?



- 6) A flagpole is anchored to the ground by a guy wire that is 12 m long. The guy wire makes an angle of 63° with the ground. How far from the base of the flagpole must the guy wire be anchored into the ground?

- 7) From the top of a 45 metre tall pole, the angle of depression to the ground is 12° . Draw a sketch to illustrate this situation, and then find the distance from the top of the pole to the ground along the sight line.

- 8) The angle of elevation of Sandra's kite string is 70° . If she has let out 55 feet of string, how high is the kite?

- 9) A cable is secured at the top of a cliff to make a zip line. If the cliff is 15 m high and the angle of depression to the zip line is 22.6° , how long does the zip line cable need to be to reach the ground?

FINDING ANGLES IN RIGHT TRIANGLES

So far in this unit, you have used the trigonometric ratios to find the length of a side. But if you know the trigonometric ratio, you can calculate the size of the angle. This requires an “inverse” operation. You can use your calculator to find the opposite of the usual ratio provided you can calculate the ratio. To do this you need 2 sides in the triangle. You can think of the inverse in terms of something simpler: addition is the opposite or inverse of subtraction. In the same way, trig functions have an inverse.

To calculate the inverse, you usually use a 2nd function and the sin/cos/tan buttons on your calculator in sequence. If you look at your calculator just above the sin/cos/tan buttons, you should see the following: \sin^{-1} , \cos^{-1} , \tan^{-1} . These are the inverse functions. If you use these buttons, you will be able to turn a ratio into an angle.

Example 1: Calculate each angle to the nearest whole degree.

a) $\sin X = 0.2546$

b) $\cos Y = 0.1598$

c) $\tan Z = 3.2785$

Solution: Use the appropriate inverse function on your calculator.

NOTE: Every calculator is different in how the buttons are keyed in order to achieve the desired outcome. Most calculators will need to key “**2ndF sin**” or “**Shift sin**” in order to get \sin^{-1} displayed. Then key in the value with or without brackets as necessary.

a) $\sin X = 0.2546$
 $X = \sin^{-1}(0.2546)$
 $X = 14.74988^{\circ}$

Angle X is 15° .

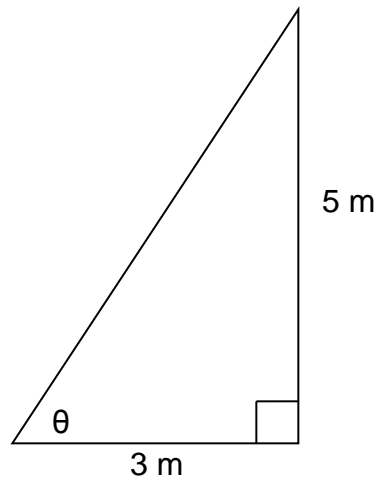
b) $\cos Y = 0.1598$
 $Y = \cos^{-1}(0.1598)$
 $Y = 80.8047^{\circ}$

Angle Y is 81° .

c) $\tan Z = 3.2785$
 $Z = \tan^{-1}(3.2785)$
 $Z = 73.03737^{\circ}$

Angle Z is 73° .

Example 2: Determine the angle θ in the following triangle.



Solution:

- 1) h, o, a the triangle
- 2) Circle the letters with their partner numbers
- 3) Choose the appropriate trig ratio. In this case, it is tangent.
- 4) Write down the ratio and fill it in.

$$\tan \theta = \frac{o}{a}$$

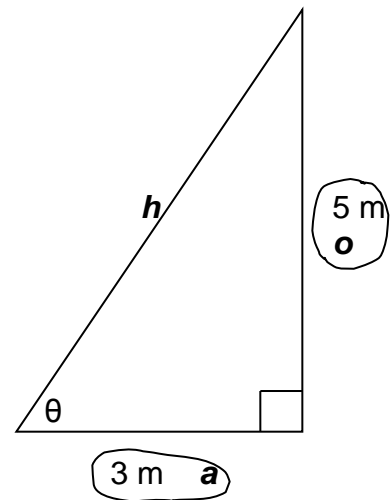
$$\tan \theta = \frac{5}{3}$$

- 5) Use the inverse function to solve for θ .

$$\theta = \tan^{-1} (5 \div 3)$$

$$\theta = 59.0352^{\circ}$$

Angle θ is approximately 59° .



NOTE – the order you use the keys is not the same on all calculators. Make sure you know how to enter this on your particular calculator!

ASSIGNMENT 10 – FINDING ANGLES IN RIGHT TRIANGLES

1) Calculate the following angles to the nearest whole degree.

a) $\sin D = 0.5491$

b) $\cos F = 0.8964$

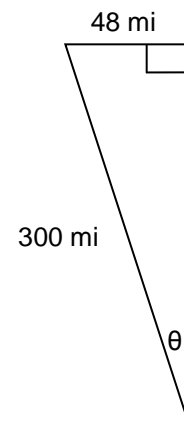
c) $\tan G = 2.3548$

d) $\sin P = 0.9998$

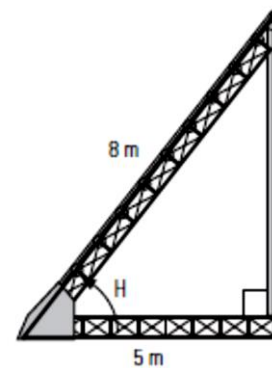
e) $\cos Q = 0.3097$

f) $\tan R = 0.4663$

2) After an hour of flying, a jet has travelled 300 miles, but gone off course 48 miles west of its planned flight path. What angle, θ , is the jet off course?



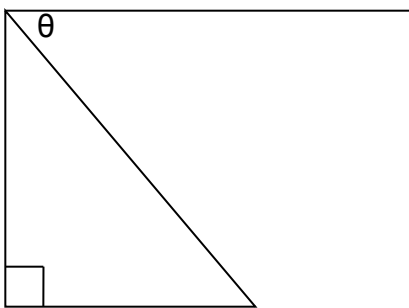
3) At what angle to the ground is an 8 m long conveyor belt if it is fastened 5 m from the base of the loading ramp?



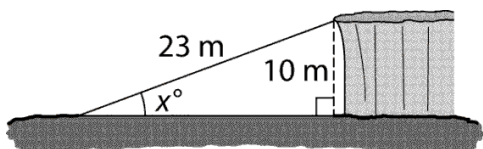
4) If a boat is 150 m from the base of a 90 m cliff, what is the angle of elevation from the boat to the top of the cliff?

5) A statue of Smokey the Bear is found in Revelstoke, BC. At a distance of 6.3 m from the base, the angle of elevation to the top is 55° . How tall is the statue?

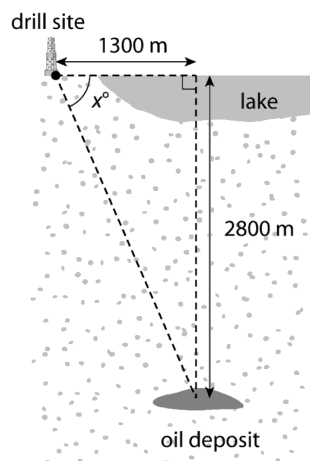
6) What is the angle of depression, θ , from the top of a 65 m cliff to an object 48 m from its base?



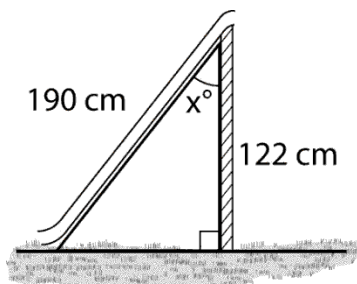
- 7) A cable is secured at the top of a cliff to create a zip line. What angle, x° , does the zip line make with the ground, to the nearest whole degree?



- 8) Justin works for an oil company. He needs to drill a well to make an oil deposit below the surface of the lake. The drill site is located on land as shown. What is the angle of depression, x° , for drilling the well? Round to the whole degree.



- 9) What angle, x° , does the slide meet the ladder? Round to the nearest whole degree.

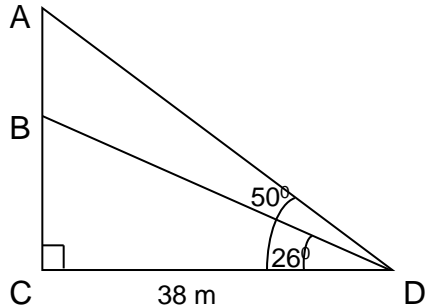


ASK YOUR TEACHER FOR QUIZ 3

SOLVING COMPLEX PROBLEMS

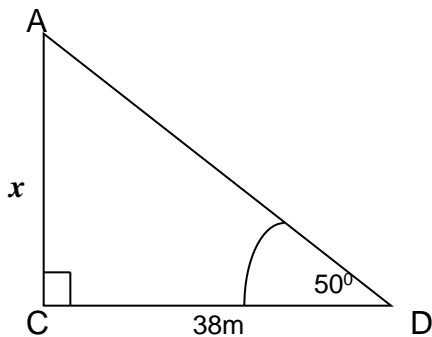
In some circumstances, you will have two or more triangles together in one diagram, and you will need to complete several steps in order to find the answer for the angle or the side you are specifically asked for. These multi-step problems are no harder than a single triangle problem as long as you follow through with the method you have been taught.

Example: In the following diagram, find the length of AB.



NOTE: The way the two (or three) triangles are arranged will not always be the same as shown in this example. It is helpful to draw the individual triangles and work with them separately.

Solution: Find the lengths of AC and BC using the appropriate trig ratio. Then subtract to find AB.



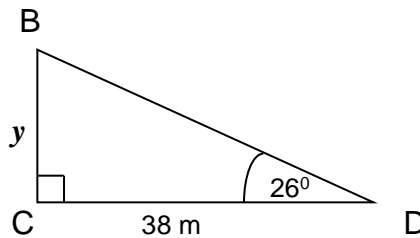
$$\tan \theta = \frac{o}{a}$$

$$\tan 50^\circ = \frac{x}{38}$$

$$x = \tan 50^\circ \times 38$$

$$x = 45.3 \text{ m}$$

$$\text{So } AB = AC - BC = 45.3 - 18.5 = 26.8 \text{ m}$$



$$\tan \theta = \frac{o}{a}$$

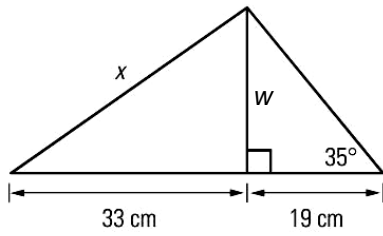
$$\tan 26^\circ = \frac{y}{38}$$

$$y = \tan 26^\circ \times 38$$

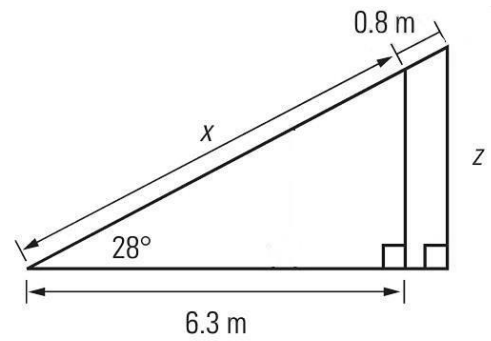
$$y = 18.5 \text{ m}$$

ASSIGNMENT 11 – WORKING WITH TWO TRIANGLES

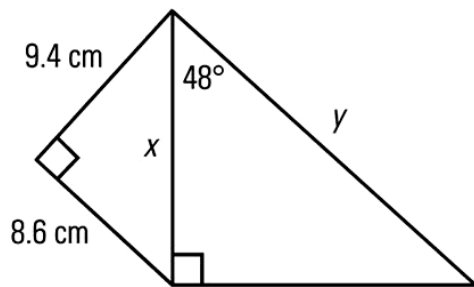
1) What is the length of x in the diagram below? Hint: Find w first and use it to find x ..



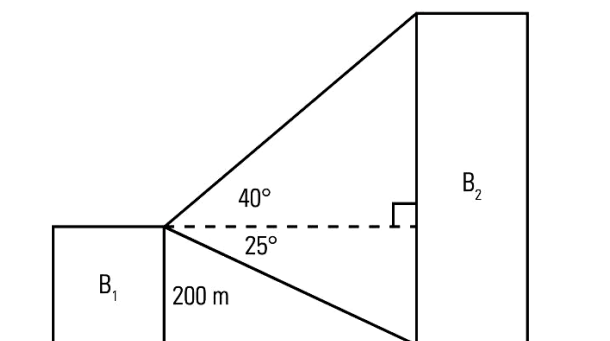
2) Find the lengths of x and z below.



3) Find the lengths of x and y below. Hint: use Pythagorean Theorem to find x first.

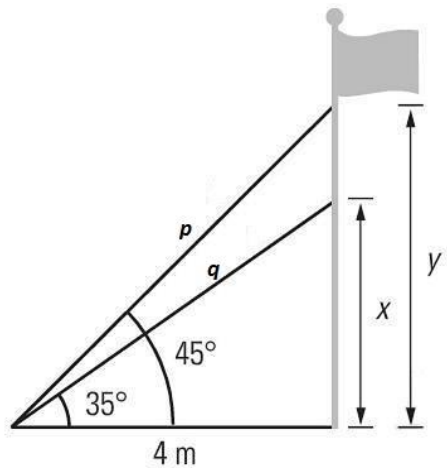


4) From the top of 200 m tall office building (B_1), the angle of elevation to the top of another building (B_2) is 40° . The angle of depression to the bottom of that building is 25° . How tall is that second building (B_2)?



5) A flagpole is supported by two guy wires, each attached to the same peg in the ground that is 4 m from the base of the flagpole. The guy wires have angles of elevation of 35° and 45° as shown below.

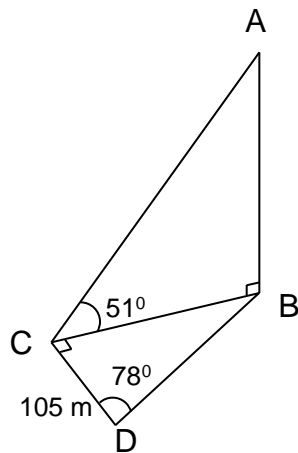
- How long is each guy wire, lengths p and q ?
- How much higher up the flagpole is the top guy wire attached? To do this, calculate the heights x and y and then calculate the difference between them.



SOLVING COMPLEX 3D PROBLEMS IN THE REAL WORLD

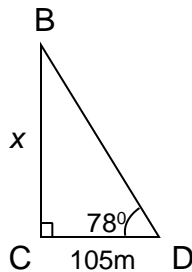
In some situations, you will need to work with triangles that are at an angle to each other. Also, some situations will involve triangles that share a common edge but in each triangle this edge will represent a different dimension. It is sometimes hard to visualize these diagrams as they are trying to represent three dimensional images on a two dimensional paper. If you are having difficulties, draw the triangles separately and work that way. Remember, we are **always** using right triangles in these problems.

Example: Calculate the height of a cliff, AB below, given the information on the diagram.



Solution: Use $\triangle BCD$ to find side BC, and then use that length in $\triangle ABC$ to find AB

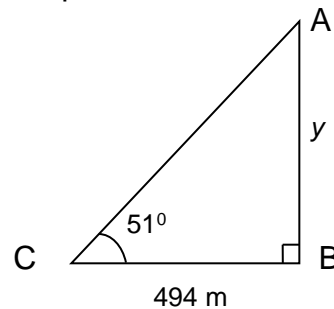
Step 1: $\triangle BCD$



$$\begin{aligned}\tan \theta &= \frac{o}{a} \\ \tan 78^\circ &= \frac{x}{105} \\ x &= \tan 78^\circ \times 105 \\ x &= 493.986 \dots \text{ m} \\ x &= 494 \text{ m}\end{aligned}$$

Now use this length in the second triangle.

Step 2: $\triangle ABC$

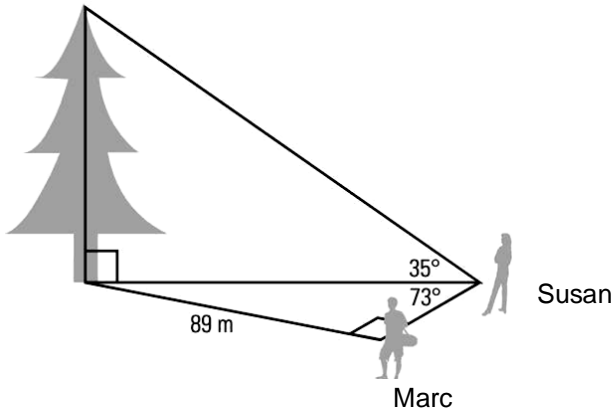


$$\begin{aligned}\tan \theta &= \frac{o}{a} \\ \tan 51^\circ &= \frac{y}{494} \\ y &= \tan 51^\circ \times 494 \\ y &= 610.0 \text{ m}\end{aligned}$$

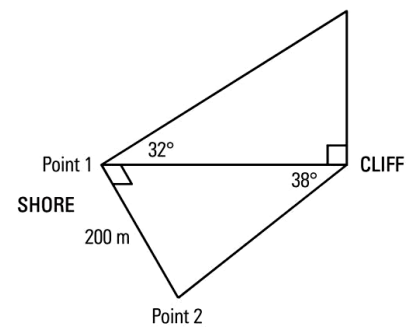
The height of the cliff, AB = 610 m

ASSIGNMENT 12 – WORKING WITH TRIANGLES IN 3-D

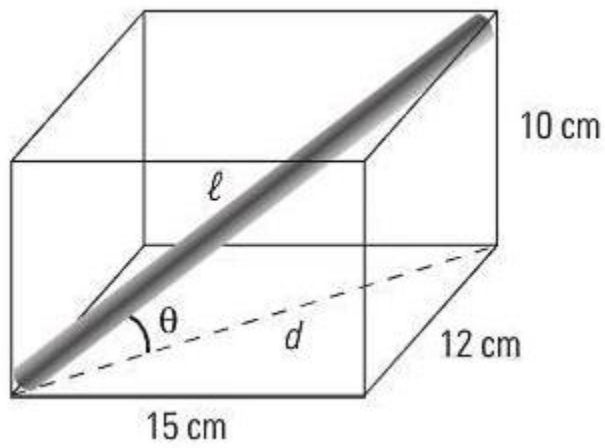
- 1) Susan and Marc spot a bird's nest at the top of a tree. Marc is 89 m from the tree. The angle between Susan's line of sight and Marc's line of sight is 73° . If the angle of elevation from Susan to the top of the tree is 35° , what is the height of the nest in the tree – how tall is the tree?



- 2) You need to calculate the height of a cliff that drops vertically into a river. Use the information in the diagram to calculate the height of the cliff.



3) A box (shown below) is 10 cm by 12 cm by 15 cm. Length d is the diagonal along the bottom of the box.



a) What is the length of the longest rod that can be carried in this box?

b) What angle, θ , does it make with the bottom of the box?